APPROXIMATE SOLUTION OF THE MHD BOUNDARY LAYER NEWTONIAN FLUIDS OVER CONTINUOUS MOVING SURFACE

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***Abstract*- To solve the equation of of laminar boundary layer flow for Newtonian power-law fluid over a continuous moving surface in the presence of transverse magnetic field. The governing nonlinear equations and their associated boundary conditions are transformed into linear differential equation and solve it using quartic spline collocation method. Table and Graphical presentation of flow problem is also given.**

***Keywords:* Quartic Spline collocation, Ordinary differential equation, Quasilinearization, Upper triangular matrix, Linear equations.**

1 INTRODUCTION

Many branches of engineering in recent years. For examples, in the extrusion of polymer sheet from a die, the lamination and melt-spinning process in the extrusion of polymers or the cooling of a large metallic plate in a bath, glass blowing continuous casting and spinning of fibers Sakiadis [1-3] studied the boundary layer behavior on a continuous solid surface moving on both flat and the cylindrical surface. Wu [4] presented the effects of suction or injection in a steady two-dimensional MHD boundary layer flow of on a flat plate. Takhar et al [5] obtained MHD asymmetric flow over a semi-infinite moving surface and numerical solution.[6] Jean-David Hoernel (2008) has been investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow.[7] Govind R. Rajput et al (2014) studied Group Theoretic Treatment.

The governing equations here are highly nonlinear differential equations, which are solved by using the Quartic spline collocation method. In this way, the paper has been organized as follows. In section 2, we use the Quartic spline collocation method. Section 3, approximate solution for the governing equations and contains, the results and discussion are summarized in section 4.

2. QUARTIC SPLINE COLLOCATION METHOD

Consider equally spaced knots of partition π: $a = x_0 < x_1 < x_2 < \dots\dots < x_n = b$ on $[a,b]$.[8] The quartic spline is defined by

$$s(x) = a_0 + b_0(x-x_0) + \frac{1}{2}c_0(x-x_0)^2 + \frac{1}{6}d_0(x-x_0)^3 + \frac{1}{24}\sum_{k=0}^{n-1} e_k (x-x_k)_+^4 \qquad (1)$$

Where the powers function $(x-x_k)_+$ is defined as

$$(x-x_k)_+ = \begin{cases} x-x_k, & x > x_k \\ 0, & x \le x_k \end{cases} \qquad (2)$$

and the boundary value problem is given by

$$y'''(x)+p(x)\,y''(x)+q(x)\,y'(x)+r(x)\,y(x)=m(x) \tag{3}$$

Subject to boundary conditions

$$\alpha_0\, y_0 \,+\, \beta_0\, y_n\,' + \gamma_0\, y_n\,'' = \delta_0$$
$$\alpha_1\, y_0\,' \,+\, \beta_1\, y_n \,+ \gamma_1\, y_n\,'' = \delta_1$$
$$\alpha_2\, y_0\,'' \,+\, \beta_2\, y_n \,+ \gamma_2\, y_n\,' = \delta_2$$

To solve this boundary value problem substitute $s(x)$, $s'(x)$, $s''(x)$, $s'''(x)$ from quartic spline, then the boundary value problem becomes

$$\sum_{k=0}^{n-1} e_k \left\{ \left(x_i - x_k\right)_+ + \frac{1}{2} p_i \left(x_i - x_k\right)_+^2 + \frac{1}{6} q_i \left(x_i - x_k\right)_+^3 + \frac{1}{24} r_i \left(x_i - x_k\right)_+^4 \right\}$$

$$+ d_0 \left\{ 1 + p_i \left(x_i - x_0\right) + \frac{1}{2} q_i \left(x_i - x_0\right)^2 + \frac{1}{6} r_i \left(x_i - x_0\right)^3 \right\}$$

$$+ c_0 \left\{ p_i + q_i \left(x_i - x_0\right) + \frac{1}{2} r_i \left(x_i - x_0\right)^2 \right\} \tag{4}$$

$$+ b_0 \left\{ p_i + r_i \left(x_i - x_0\right) \right\} + a_0 \left\{ r_i \right\} = m\left\{ x_i \right\}$$ .Where i = 0,1,2,.....n.

Thus for quartic spline and third order boundary value problem we get nine linear algebraic equations in nine unknowns a_0, b_0, c_0, d_0, e_0, e_1,.... e_4. The matrix form of this system is given by

$AX = B$

Where $X = [e_4, e_3, e_2, e_1, e_0, d_0, c_0, b_0, a_0]^T$

$$B = \left[\delta_2, \delta_1, \delta_0, m_5, m_4, m_3, m_2, m_1, m_0\right]^T$$

And the co-efficient matrix A is an upper Hessenberg matrix.

Quartic spline and third order boundary value problem, we take number of intervals n=5. Continuing this process we can say that "In general for higher degree spline and lower order boundary value problem i.e. for n^{th} degree spline and $(n-1)^{th}$ order boundary value problem we can take (n+1) number of intervals at x_i, $i = 0(1)\, n$ and get a set of linear algebraic equations in n+4 unknowns."

3. SOLUTION BY USING COLLOCATION METHOD

[9]Srivastava et al (1987), the basic equations governing the motion of two dimensional, steady incompressible viscous fluids past continuous surface in the presence of transverse magnetic field can be written in non-linear coupled equation as follow:

$$f''' - \beta f' + \frac{1}{2} ff'' = 0, \tag{5}$$

If $\beta = 0$, i.e. for non-magnetic case. Subject to boundary conditions as,

$$f'(0) = 0.5,\ f(0) = 0,\ f'(1) = 1. \tag{6}$$

We use quasilinearization technique to convert (5) into linear form with help of boundary conditions (6). We get linear form as

$$f_{i+1}^{'''} + \frac{1}{2} f_i f_{i+1}^{''} - \beta f_{i+1}^{'} + \frac{1}{2} f_i^{''} f_{i+1} = \frac{1}{2} f_i^{''} f_i \tag{7}$$

With boundary conditions as (6).The Quartic spline is given by

$$s(\eta) = a_0 + b_0(\eta - \eta_0) + \frac{1}{2} c_0 (\eta - \eta_0)^2 + \frac{1}{6} d_0 (\eta - \eta_0)^3 + \frac{1}{24} \sum_{k=0}^{n-1} e_k (\eta - \eta_k)_+^4 \tag{8}$$

Substitute (8) in (7) and use assume curve with given boundary conditions we get collocation as follows:

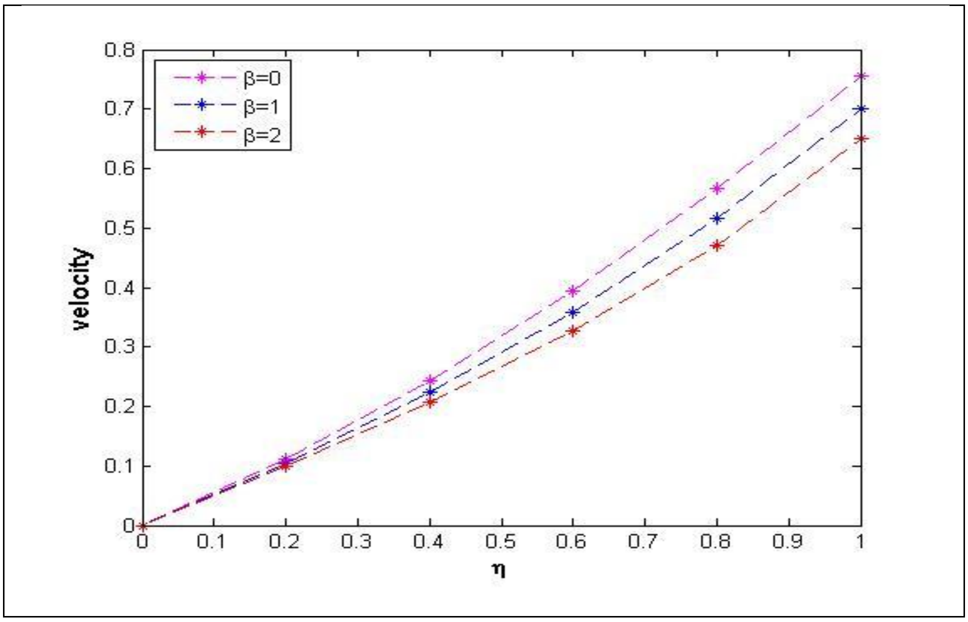
$$\begin{aligned}
&\sum_{k=0}^{n-1} e_k [(\eta_i - \eta_k) + \frac{f_i}{2}(\eta_i - \eta_k)^2 - \frac{\beta}{6}(\eta_i - \eta_k)^3 + \frac{1}{96}(\eta_i - \eta_k)^4] \\
&+ d_0[1 + f_i(\eta_i - \eta_0) - \frac{\beta}{2}(\eta_i - \eta_0)^2 + \frac{1}{24}(\eta_i - \eta_0)^3] \\
&+ c_0[f_i - \beta(\eta_i - \eta_0) + \frac{1}{8}(\eta_i - \eta_0)^2] \\
&+ b_0[-\beta + \frac{1}{4}(\eta_i - \eta_0)] \\
&+ a_0[\frac{1}{4}] = \frac{1}{8} f_i
\end{aligned} \tag{9}$$

Solve above equation and substitute constants in (7) and we get the solution for different values of β .

Table: solution of problem using Numerical method:

	Numerical solution with spline		
η	S(η) for $\beta = 0$	S(η) for $\beta = 1$	S(η) for $\beta = 2$
0.0	0	0	0
0.2	0.1105184	0.105007	0.100289839
0.4	0.241956854	0.222845013	0.206566372
0.6	0.393948767	0.357989294	0.32726458
0.8	0.5658509	0.515272947	0.471607739
1.0	0.756732427	0.700185353	0.649992345

Graphical solution of given problem



4. RESULT AND DISCUSSION

The Graphical representation shows that, there is a significant impact of magnetic field on the velocity profile of the flow. Here we find velocity profile for different values of β . From Figure it is clear that Value of β increase the velocity of fluid decrease. It means that magnetic field increase, velocity of fluid flow decrease.

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