UNSTEADY MHD TWO-FLUID FLOW AND HEAT TRANSFER THROUGH A HORIZONTAL CHANNEL

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***ABSTRACT*- An unsteady MHD flow of two viscous immiscible incompressible electrically conducting fluids and heat transfer through a horizontal channel with isothermal non-conducting permeable plates in the presence of transverse magnetic field is investigated. The partial differential equations governing the flow and heat transfer are transformed to ordinary differential equations and closed-form solutions are obtained in both fluids. Numerical results for velocity and temperature fields are presented graphically and the numerical values of skin-friction and Nusselt number have been tabulated. The effect of different parameters like viscosity ratio, phase angle, Hall parameter, thermal conductivity ratio and Prandtl number on velocity field, temperature field, skin friction and Nusselt number are discussed.**

***Keywords:* MHD, isothermal, electrically conducting fluid, skin friction, Nusselt number**

I INTRODUCTION

Study of flow through and past a porous medium constitutes a comparatively recent development in fluid mechanics, with applications in Science, Engineering and Technology. More specifically, the existence of a fluid layer adjacent to a layer of fluid saturated porous medium is a common occurrence in both geophysical and engineering environments. Modeling of such systems requires understanding of the convective interaction between the fluid layer and the adjacent fluid saturated porous medium. Some of the geothermal sources e.g. One in the Gulf of California, lie very close to the surface and under a body of water. These situations may be molded as fluid superposed, porous layers.

Packham and Shail (1971) studied stratified laminar flow of two immiscible fluids. Soundalgekar and Bhat(1971) considered oscillatory MHD flow and heat transfer through a channel. Magnetohydrodynamic heat transfer in two phase flow between parallel plates was discussed by Lohrasbi and Sahai (1988). Zaturska et al. (1988) analyzed flow of viscous fluid driven along a channel by suction at porous walls. Two-dimensional flow of a viscous fluid in a channel with porous walls was considered by Cox (1991). Malashetty and Leela (1991) investigated magneto hydrodynamic heat transfer in two fluid flow. Malashetty and Leela (1992) studied magneto hydrodynamic heat transfer in two phase of flow. Berman (1993) investigated laminar flow in channels with porous walls. Attaia and Kotb (1996) considered MHD flow between two parallel plates with heat transfer. Malashetty and Chamkha (2000) reported analytical solutions for flow of two immiscible fluids in porous and non-porous channels. Malashetty et al. (2004) studied fully developed flow and heat transfer in a horizontal channel containing an electrically conducting fluid sandwiched between two fluid layers. Umawathi et al. (2004) considered fully developed flow and heat transfer in a horizontal channel containing an electrically conducting fluid sandwiched between two fluid layers. Umawathi and Mateen (2004) analyzed unsteady two-fluid flow and heat transfer in a horizontal channel. Pilecks and Socolowsky(2005) considered viscous two-fluid flows in perturbed unbound domains. Umawathi et al. (2006) analyzed oscillatory Hartman two-fluid flow and heat transfer in a horizontal channel. Tsuyoshi and Shu-Ichiro (2008) studied two-fluid magneto hydrodynamic simulation of converging hi flows in the interstellar medium. Linga and Sreedhar (2009) analyzed unsteady two-fluid flow and heat transfer of conducting fluids in channels under transverse magnetic field. Magnetic fields for fluid motion were discussed by Weston et al (2010). Kumar et al (2012) discussed the unsteady MHD free convective flow through porous medium sandwiched between viscous fluids.

II FORMULATION OF THE PROBLEM

Consider a two-dimensional unsteady flow of two viscous immiscible incompressible electrically conducting fluids through horizontal parallel permeable non-conducting plates, extending in the x^*- and y^*- directions in the presence of transverse magnetic field of uniform intensity B_0 region – I($o \le y^* \le h$) is filled with a viscous incompressible fluid having density ρ_1, dynamic viscosity μ_1, specific heat at constant pressure C_{p1}, thermal conductivity κ_1 and region -II ($-h \le y \le 0$) is filled with a different viscous incompressible fluid having density ρ_2, dynamic viscosity μ_2, specific heat at constant pressure C_{p2} and thermal conductivity κ_2.

The flow in both regions of the channel is assumed to be fully developed and is driven by a common pressure gradient $\left(-\frac{\partial p}{\partial x}\right)$. Both the plates are maintained at isothermal temperatures T_{w1}, T_{w2} at y^*= h, y^*= -h, respectively. All fluid properties are assumed to be constant.

Under these assumptions, taking $\rho_1 = \rho_2 = \rho$ and $C_{p1}=C_{p2}=C_p$, the governing equations of motion and energy with Hall current effects [Jeffery(1961), Bansal (1994) etc.] are given by

Region-1

$$\frac{\partial v_1^*}{\partial y^*}=0, \quad (1)$$

$$\rho\left(\frac{\partial u_1^*}{\partial t^*}+v_1^*\frac{\partial u_1^*}{\partial y^*}\right)=-\frac{\partial p}{\partial x}+\mu_1\frac{\partial^2 u_1^*}{\partial y^{*2}}-\sigma B_0^2 u_1^*, \quad (2)$$

$$\rho C_p\left(\frac{\partial T_1^*}{\partial t^*}+v_1^*\frac{\partial T_1^*}{\partial y^*}\right)=\kappa_1\frac{\partial^2 T_1^*}{\partial y^{*2}}+\mu_1\left(\frac{\partial u_1^*}{\partial y^*}\right)^2+\sigma B_0^2 u_1^{*2} \quad (3)$$

Region-II

$$\frac{\partial v_2^*}{\partial y^*}=0, \quad (4)$$

$$\rho\left(\frac{\partial u_2^*}{\partial t^*}+v_2^*\frac{\partial u_2^*}{\partial y^*}\right)=-\frac{\partial p}{\partial x}+\mu_2\frac{\partial^2 \mu_2^*}{\partial y^{*2}}-\sigma B_0^2 u_2^*, \quad (5)$$

$$\rho C_p\left(\frac{\partial u_2^*}{\partial t^*}+v_2^*\frac{\partial u_2^*}{\partial y^*}\right)=k_2\frac{\partial^2 T_2^*}{\partial y^{*2}}+\mu_2\left(\frac{\partial u_2^*}{\partial y^*}\right)^2+\sigma B_0^2 u_2^{*2}, \quad (6)$$

where u^* denotes component of fluid velocity along x^* -direction. v^* denotes component of fluid velocity along y^* -direction and T^* is the fluid temperature. The subscripts 1 and 2 correspond to Region-I and Region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which require that the x^*-component of velocity must vanishes at the wall. The boundary conditions on temperature at the walls are isothermal conditions. The continuity of velocity, shear stress, temperature and heat flux at the interface between the fluid layers at $y^*=0$ is considered.

The hydrodynamic boundary and interface conditions for the two-fluids are given by

$$y^*=-h: u_2^0=0, v_2^*=v^*\left(t^*\right), T_2^*=T_{w2}$$

$$y^*=0: u_1^*=u_2^*, \mu_1\frac{\partial u_1^*}{\partial y^*}=\mu_2\frac{\partial u_2^*}{\partial y^*}, T_1^*=T_2^*,$$

$$\kappa_1\frac{\partial T_1^*}{\partial y^*}=\kappa_2\frac{\partial T_2^*}{\partial y^*};$$

$$y^*=+h: u_1^*=0, v_1^*=v^*\left(t^*\right), T_1^*=T_{w1}. \quad (7)$$

The continuity equations of both fluids imply that v_1^* and v_2^* are independent of y^* and they can be at most function of time alone.

Here, it is assumed that the transpiration velocity v^* varies periodically with time about a non-zero constant mean V_0 i.e. $v^*=V_0\left(1+\in Ae^{i\omega^* t^*}\right)$,

where $v_1^*=v_2^*=v^*$, A is a real positive constant, ω^* is a frequency parameter and $\in$ is small parameter such that $0 < \varepsilon A \le 1$.

III METHOD OF SOLUTION

Introducing the following non-dimensional quantities

$$y^*=hy, t^*=\frac{h^2}{\nu_1}t, u_1^*=\bar{u}_1 u_1, v^*=vV_0,$$

$$\theta_1=\frac{T_1^*-T_{w2}}{\left(\frac{\overline{u_1^2\mu_1}}{\kappa_1}\right)}, \omega^*=\frac{\nu_1}{h^2}\omega, P=\frac{-h^2}{\mu_1\overline{\mu_1}}\left(\frac{\partial p}{\partial x}\right), \mu_r=\frac{\mu_2}{\mu_1},$$

$$\kappa_r=\frac{k_2}{k_1}, T_{w1}-T_{w2}=\frac{\overline{u_1^2\mu_1}}{\kappa_1}, P_r=\frac{\mu_1 C_p}{\kappa_1}, M=B_0 h\sqrt{\frac{\sigma}{\mu_1}} \quad (8)$$

Into the equations (2), (3), (5) and (6), we get

Region-I

$$\frac{\partial u_1}{\partial t}+v\frac{\partial u_1}{\partial y}=\frac{\partial^2 u_1}{\partial y^2}+P-M^2 u_1 \quad (9)$$

$$\frac{\partial \theta_1}{\partial t}+v\frac{\partial \theta_1}{\partial y}=\frac{1}{\Pr}\left\{\frac{\partial^2\theta_1}{\partial y^2}+\left(\frac{\partial u_1}{\partial y^2}\right)^2\right\}+\frac{M^2 u_1^2}{\Pr} \quad (10)$$

Region-II

$$\frac{\partial u_2}{\partial t}+v\frac{\partial u_2}{\partial y}=\mu_r\frac{\partial^2 u_2}{\partial y^2}+P-M^2 u_2, \quad (11)$$

$$\frac{\partial \theta_2}{\partial t}+v\frac{\partial \theta_2}{\partial y}=\frac{\kappa_r}{P_r}\frac{\partial^2\theta_2}{\partial y^2}+\frac{\mu_r}{\Pr}\left(\frac{\partial u_2}{\partial y}\right)^2+\frac{M^2 u_2^2}{\Pr}, \quad (12)$$

where Pr is the Prandtl number, μ_r is the ratio of viscosities, k_r is the ratio of thermal conductivities and M is Hartman number.

The corresponding boundary conditions are reduced to

$$y=-1: u_2=0, \theta_2=0;$$

$$y=0: u_1=u_2, \frac{\partial u_1}{\partial y}=\mu_r\frac{\partial u_2}{\partial y}, \theta_1=\theta_2, \frac{\partial \theta_1}{\partial y}=\kappa_r\frac{\partial \theta_2}{\partial y};$$

$$y=+1: u_1=0, \theta_1=0. \quad (13)$$

Equations (9) to (12) are coupled partial differential equations. The velocity and temperature distributions are separated into steady and unsteady parts as given below

$$F(y,t)=F_0(y)+\in e^{i\omega t}F_1(y), \quad (14)$$

where F stands for $u_1, u_2, \theta_1 or\, \theta_2$ .Substituting equation (14) into the equations (9) to (12) and equating the harmonic and non-harmonic terms, we get

Zeroth-order Equations
Region-I

$$\frac{d^2u_{10}}{dy} - \frac{du_{10}}{dy} - M^2u_{10} = -P\,, \tag{15}$$

$$\frac{d^2\theta_{10}}{dy^2} - \Pr\frac{d\theta_{10}}{dy} = -\left(\frac{du_{10}}{dy}\right)^2 - M^2u_{10}^2\,, \tag{16}$$

Region-II

$$\mu_r\frac{d^2u_{20}}{dy^2} - \frac{du_{20}}{dy} - M^2u_{20} = -P\,, \tag{17}$$

$$\kappa_r\frac{d^2\theta_{20}}{dy^2} - \Pr\frac{d\theta_{20}}{dy} = -\mu_r\left(\frac{du_{20}}{dy}\right)^2 - M^2u_{20}\,, \tag{18}$$

First Order Equations
Region-I

$$\frac{d^2u_{11}}{dy^2} - \frac{du_{11}}{dy} - \left(M^2 + i\omega\right)u_{11} = A\frac{du_{10}}{dy}\,, \tag{19}$$

$$\frac{d^2\theta_{11}}{dy^2} - \Pr\frac{d\theta_{11}}{dy} - i\omega\Pr\theta_{11} = A\Pr\frac{d\theta_{10}}{dy}\,,$$
$$-2\frac{du_{10}}{dy}\frac{du_{11}}{dy} - 2M^2u_{10}u_{11} \tag{20}$$

Region-II

$$\mu_r\frac{d^2u_{21}}{dy^2} - \frac{du_{21}}{dy} - \left(M^2 + i\omega\right)u_{21} = A\frac{du_{20}}{dy}\,, \tag{21}$$

$$K_r\frac{d^2\theta_{21}}{dy^2} - \Pr\frac{d\theta_{21}}{dy} - \left(M^2 + i\omega\right)u_{21} = A\frac{du_{20}}{dy}\,, \tag{22}$$

The corresponding boundary conditions are reduced to

$$y = -1: u_{20} = 0, u_{21} = 0, \theta_{20} = 0, \theta_{21} = 0\,;$$

$$y = 0: u_{10} = u_{20}, \frac{du_{10}}{dy} = \mu_r\frac{du_{20}}{dy}, u_{11} = u_{21},$$

$$\frac{du_{11}}{dy} = \mu_r\frac{du_{21}}{dy},$$

$$\theta_{10} = \theta_{20}, \frac{d\theta_{10}}{dy} = \kappa_r\frac{d\theta_{20}}{dy}, \theta_{11} = \theta_{21}, \frac{d\theta_{11}}{dy} = \kappa_r\frac{d\theta_{21}}{dy};$$

$$y = 1: u_{10} = 0, u_{11} = 0, \theta_{10} = 1, \theta_{11} = 0\,. \tag{23}$$

Equations (15) to (22) are ordinary differential equations and solved under the boundary conditions (23) through straight forward calculations, the solution $u_{10}(y), u_{11}(y), u_{20}(y)$, $\theta_{10}(y), \theta_{11}(y), \theta_{20}(y)$ and $\theta_{21}(y)$, are known and given by

$$u_{10}(y) = C_1e^{m_1y} + C_2e^{m_2y} + \frac{P}{M^2}\,, \tag{24}$$

$$u_{11}(y) = C_5e^{(l_1+il_2)y} + C_6e^{(l_3-il_2)y}$$
$$+\frac{iA}{\omega}\left(C_1m_1e^{m_1y} + C_2m_2e^{m_2y}\right) \tag{25}$$

$$u_{20}(y) = C_3e^{m_3y} + C_4e^{m_4y} + \frac{P}{M^2}\,, \tag{26}$$

$$u_{21}(y) = C_1e^{(l_4+il_5)} + C_8e^{(l_6-il_5)y}$$
$$+\frac{iA}{\omega}\left(C_3m_3e^{m_3y} + C_4m_4e^{m_4y}\right) \tag{27}$$

$$\theta_{10}(y) = C_9 + C_{10}e^{\Pr y} - A_{11}e^{2m_1y} - A_{12}e^{2m_2y}\,,$$
$$-A_{13}e^{(m_1+m_2)y} - A_{14}e^{m_1y} - A_{15}e^{m_2y} + \frac{P^2y}{\Pr M^2} \tag{28}$$

$$\theta_{11}(y) = C_{13}e^{(l_{47}+il_{48})y} + C_{14}e^{(l_{49}-il_{48})y} - il_{56}$$
$$+il_{55}e^{\Pr y} - (l_{119}+il_{120})e^{2m_1y} - (l_{121}+il_{122})e^{2m_2y} - (l_{123}+il_{124})e^{(m_1+m_2)y}$$
$$-(l_{125}+il_{126})e^{m_1y} - (l_{127}+il_{128})e^{m_2y}$$
$$-2(l_{129}+il_{130})e^{(m_1+l_1+il_2)y} - 2(l_{131}+il_{132})e^{(m_1+l_3-il_2)y}$$
$$-2(l_{133}+il_{134})e^{(m_2+l_1+il_2)y} - 2(l_{135}+il_{136})e^{(m_2+l_3-il_2)y}$$
$$-2P(l_{115}+il_{116})e^{(l_1+il_2)y} - 2P(l_{117}+il_{118})e^{(l_3-il_2)y} \tag{29}$$

$$\theta_{20}(y) = C_{11} + C_{12}e^{(\Pr/k_r)y} - A_{16}e^{2m_3y} - A_{17}e^{2m_4y}$$
$$-A_{18}e^{(m_3+m_4)y} - A_{19}e^{m_3y} - A_{20}e^{m_4y} + \frac{P^2y}{M^2} \tag{30}$$

$$\theta_{21}(y) = C_{15}e^{(l_{137}+il_{138})y} + C_{16}e^{(l_{139}-il_{138})y} + il_{145}e^{(\Pr/k_r)y}$$
$$-il_{146} - (l_{213}+il_{214})e^{2m_3y} - (l_{215}+il_{216})e^{2m_4y} - (l_{217}+il_{218})e^{(m_3+m_4)y}$$
$$-(l_{219}+il_{220})e^{m_3y} - (l_{221}+il_{222})e^{m_4y}$$
$$-(l_{223}+il_{224})e^{(m_3+l_4+il_5)y} - (l_{225}+il_{226})e^{(m_3+l_6-il_5)y} - (l_{227}+il_{228})e^{(m_4+l_4-il_5)y}$$
$$-(l_{229}+il_{230})e^{(m_4+l_6-il_5)y} - 2P(l_{209}+il_{210})e^{(l_4+il_5)y}$$
$$-2P(l_{211}+il_{212})e^{(l_6-il_5)y}\,, \tag{31}$$

where $i = \sqrt{-1}$, A_1 and A_{20}, C_1 to C_{16}, l_1 to l_{230} and m_1 to m_4 are constants and their expressions are not included here for the sake of brevity.

IV SKIN- FRICTION COEFFICIENT

Skin-friction coefficient at the lower and upper plates is given by

$$C_f = \frac{\tau^*}{\rho\bar{u}_1^2/2} = \frac{2}{\text{Re}}\left(\frac{\partial u}{\partial y}\right)_{y=\pm1}\,, \tag{32}$$

Where $\tau^* = \mu_1\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=\pm h}$ and $\text{Re} = \frac{h\rho\bar{u}_1}{\mu_1}$.

Hence, skin friction coefficient at the upper plate in region – I is obtained as given below

$$(C_f)_1 = \frac{2}{\mathrm{Re}}\left(C_1 m_1 e^{m_1} + C_2 m_2 e^{m_2}\right)$$
$$+\frac{2\varepsilon}{\mathrm{Re}}\left\{\left(l_{276}\cos\omega t - l_{277}\sin\omega t\right) + i\left(l_{277}\cos\omega t + l_{276}\sin\omega t\right)\right\} \tag{33}$$

Skin friction coefficient at the lower plate in the region – II is obtained as given by

$$(C_f)_{-1} = \frac{2}{\mathrm{Re}}\left(C_3 m_3 e^{-m_3} + C_4 m_4 e^{-m_4}\right)$$
$$+\frac{2\varepsilon}{\mathrm{Re}}\left\{\left(l_{278}\cos\omega t - l_{279}\sin\omega t\right) + i\left(l_{278}\cos\omega t + l_{279}\sin\omega t\right)\right\} \tag{34}$$

Here l_{276} to l_{279} are constants and their expressions are not included here for the sake of brevity.

V NUSSELT NUMBER

Nusselt number at the lower and upper plates is given by

$$Nu = \frac{qh}{\kappa(T_{w1} - T_{w2})} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=\pm 1}, \tag{35}$$

where $q = -\kappa\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=\pm h}$.

Hence, Nusselt number at the upper plate is obtained as given below

$$(Nu)_1 = -l_{280} - \varepsilon\left\{\left(l_{281}\cos\omega t - l_{282}\sin\omega t\right)\right.$$
$$\left. + i\left(l_{282}\cos\omega t + l_{281}\sin\omega t\right)\right\} \tag{36}$$

Nusselt number at the lower plate is obtained as given below

$$(Nu)_{-1} = -l_{283} - \varepsilon\left\{\left(l_{284}\cos\omega t - l_{285}\sin\omega t\right)\right.$$
$$\left. + i\left(l_{285}\cos\omega t + l_{284}\sin\omega t\right)\right\} \tag{37}$$

Here l_{281} to l_{285} are constants and their expressions are not included here for the sake of brevity.

VI RESULT AND DISCUSSION

Figure 1 shows the immiscible fluids velocity profiles in both regions (Regions – I and Regions – II) of the channel for different values of the viscosity ratio μ_r. As μ_r (ratio of viscosity) increase, the velocity decreases in both fluid regions. As μ_r increases, the viscosity of the fluid in the lower region becomes thick and hence, the velocity decreases. Velocity decreases with the increase in Hall parameter as observed from figure 2.

The effect of phase angle on fluid velocity shows different nature in both the regions. In Region-I, fluid velocity decreases as phase angle increases upto π/4, but it increases as phase angle increases from π/2 to π in Region-II as noted from figure 3.

Fluid temperature decreases when ratio of viscosities increases in Region-I, but it increases in Region-II as seen from figure 4.

Figure 5 displays the influence of the thermal conductivity ratio κ_r on the fluid temperature profiles in both fluid regions of the channel. Increase in the thermal conductivity ratio has the tendency to cool down the thermal state in the channel.

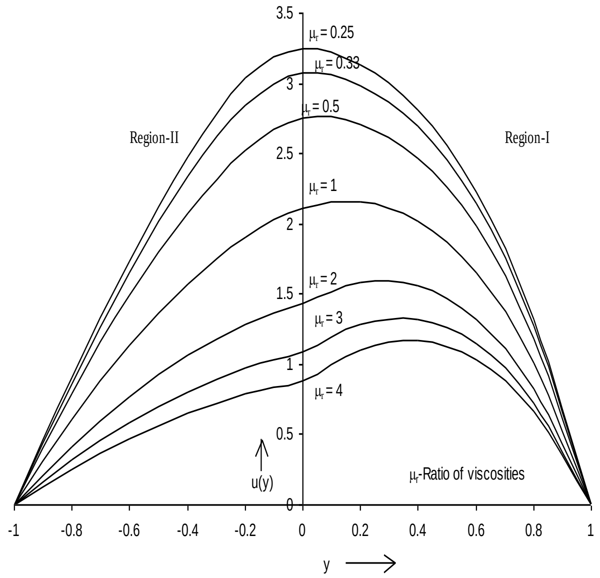
Figures 6 and 7 show that fluid temperature increases with the increase in Hall parameter and Prandtl number, respectively.

Change in phase angle shows mixed effect on fluid temperature in both the regions. In Region-I, fluid temperature increases with the increase in phase angle when $0 \le \omega t \le \pi$ and it starts decreasing when $\pi/2 \le \omega t \le \pi$. But in Region-II, fluid temperature increases when $0 \le \omega t \le \pi/4$ and it starts decreasing when $\pi/4 \le \omega t \le \pi$ as observed from fig 8.

It is noted from table – 1 that skin-fraction coefficient at the upper plate increases with the increase in Hall parameter, Prandtl number, κ_r or μ_r, while it decreases with the increase in phase angle. Skin-friction coefficient at the lower plate decreases with the increase in Hall parameter, Prandtl number or μ_r.

VII CONCLUSION

1. Fluid velocity decreases in both fluid regions due to increase in ratio of viscosities or Hall parameter.
2. Fluid Temperature decreases in both fluid regions due to increase in thermal conductivity ratio and increases with the increase in Hall parameter or Prandtl number.
3. Skin-friction coefficient increases with the increase in the ratio of thermal conductivity.
4. Nusselt number increases with the increase in Hall parameter and decreases with the increase in ratio of viscosities.



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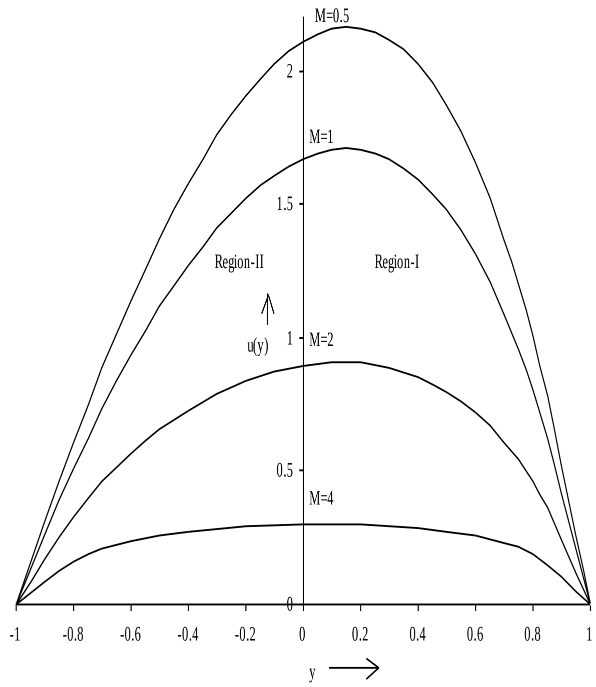
g. 1 Velocity distribution versus y when M=0.5, P=5, A=0.1, $\omega\tau = \pi/4$, Pr = 7, $\omega = 5$, $\in = 0.01$ and k_r=1.



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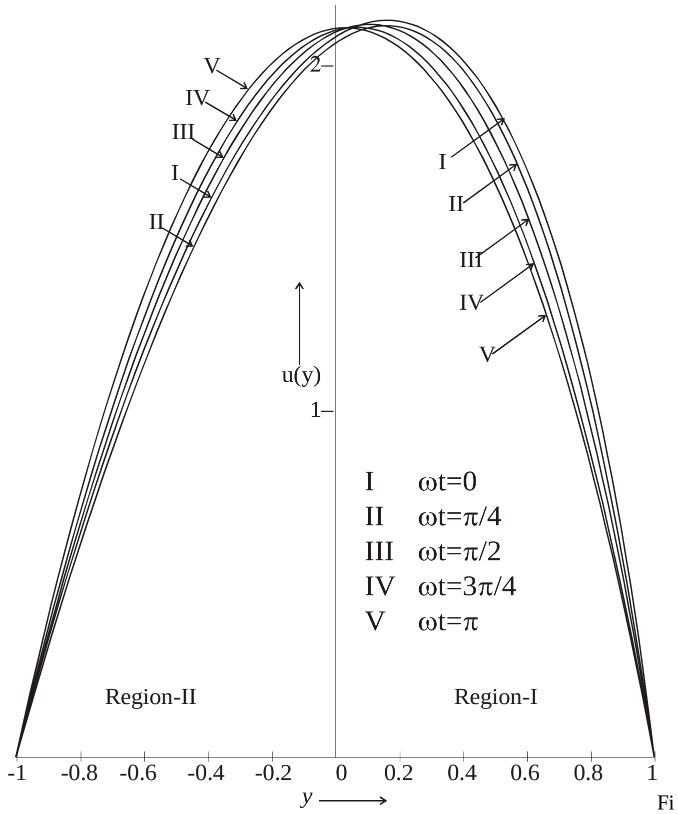
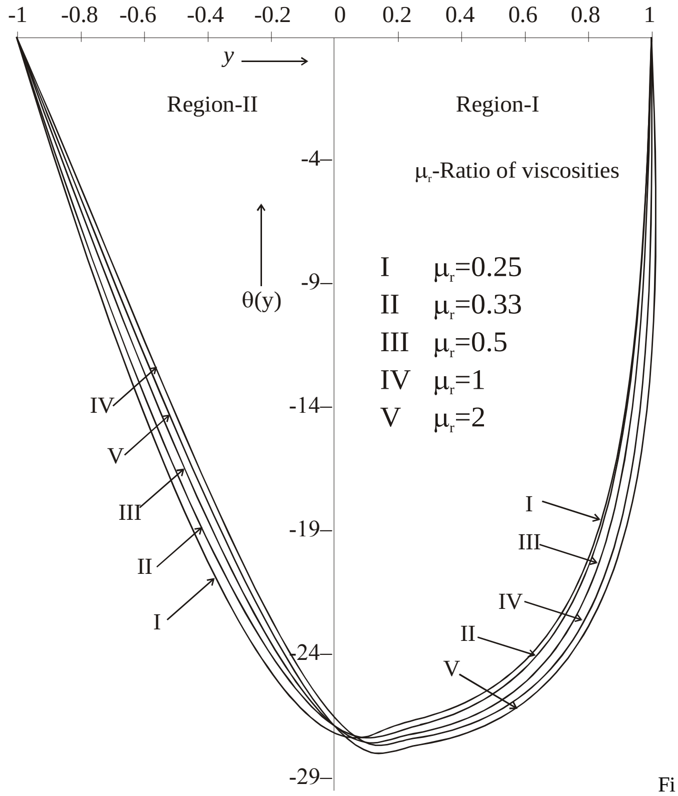
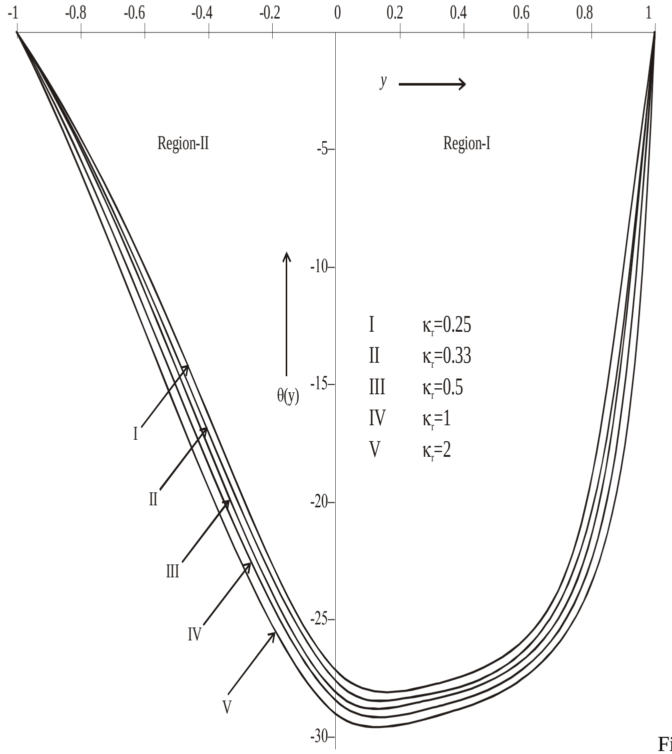
g. 2 Velocity distribution versus y when P = 5, A = 0.1, $\omega\tau = \pi/4$, Pr = 7, ω=5, $\in$=0.01, μ_r =1 and k_r =1.



Fi g. 3 Velocity distribution versus y when M=0.5, P=5, A=0.1, Pr=7, ω=5, ∈=0.01, μ_r=1 and k_r=1.



Fi g. 4 Temperature distribution versus y when P=5, A=0.1, M=0.5, α=1, ωτ= π/4, Pr=7, ω=5, ∈=0.01 and k_r=1.



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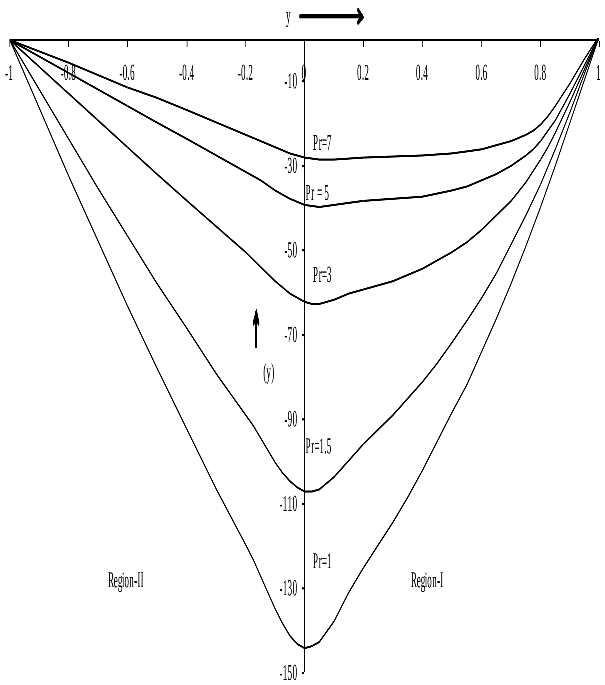
g. 5 Temperature distribution versus y when M=0.5, P=5, A=0.1, $\omega\tau = \pi/4$, Pr=7, ω=5, $\in$=0.01 and μ_r=1.



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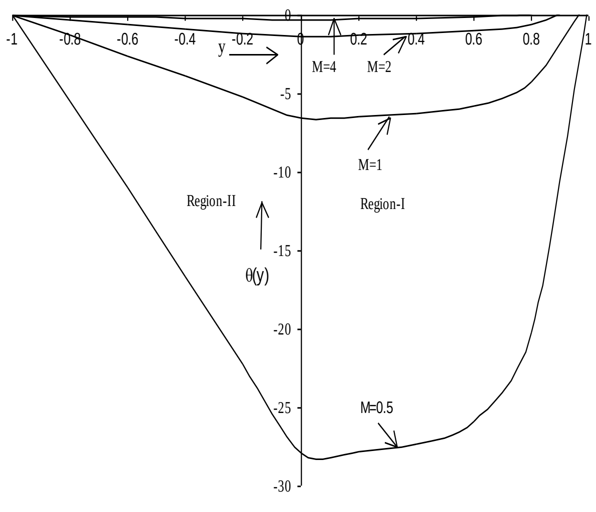
g. 7 Temperature distribution versus y when P =5, A= 0.1, M = 0.5, μ_r = 1, $\omega\tau = \pi/4$, ω = 5, $\in$ = 0.01 and k_r=1.



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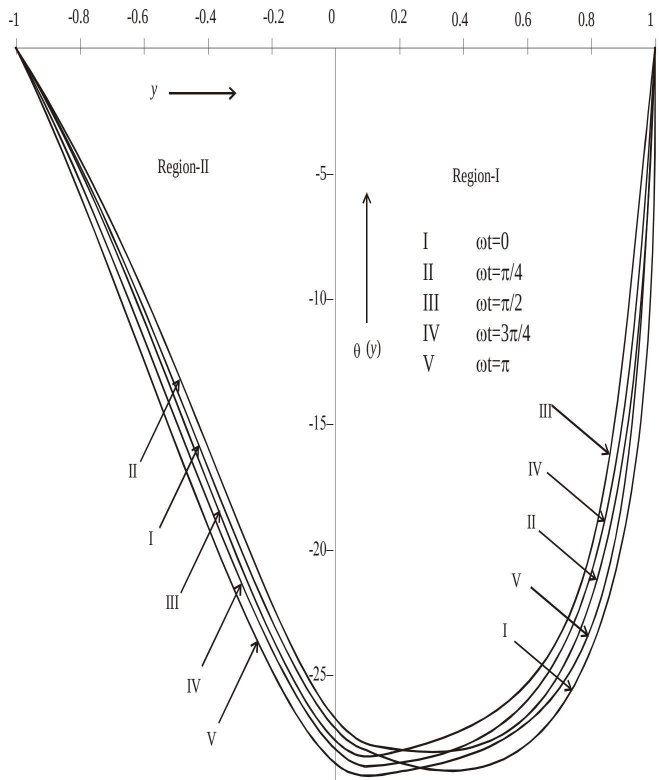
g. 6 Temperature distribution versus y when P=5, A=0.1, $\omega\tau = \pi/4$, Pr=7, ω=5, $\in$=0.01, k_r =1 and μ_r=1.



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g. 8 Temperature distribution versus y when P=5, A=0.1, M=0.5, μ_r =1, Pr =7, ω=5, $\in$=0.01 and k_r=1.

Table-1: Numerical values of skin-friction coefficient and Nusselt number at the plates for various values of physical parameters when P=5, A=0.1,ω=5 and ∈=0.01

M	Pr	κ_ρ	μ_ρ	ωτ	Skin friction coefficient		Nusselt number	
					$(C_f)_1$	$(C_f)_{-1}$	$(Nu)_1$	$(Nu)_{-1}$
0.5	7	1	1	π/4	-24.21558	12.91868	-193.76820	-87.15237
1	7	1	1	π/4	-19.73809	11.09666	-46.41765	-22.38199
0.5	5	1	1	π/4	-1210.75180	645.96502	-198.62176	-65.78132
0.5	7	0.25	1	π/4	-24.21559	12.91867	-192.84184	-87.16544
0.5	7	1	0.25	π/4	-1556.04777	931.66005	-188.10566	-85.04780
0.5	7	1	1	3π/4	-1210.74758	645.96406	-199.22585	-80.84488

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