DECOUPLED HIERARCHICAL CONTROL AND SMALL-SIGNAL STABILITY ANALYSIS OF ISLANDED INVERTER-BASED MICROGRIDS USING DIGITAL VIRTUAL IMPEDANCE

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Abstract*— This paper presents a robust hierarchical control and comprehensive state-space stability framework for inverter-based resources (IBRs) operating in autonomous islanded microgrids. In low-voltage distribution networks, highly resistive line parameters (high R/X ratio) degrade conventional active power-frequency (P-ω) and reactive power-voltage (Q-V) droop performance, inducing severe steady-state and transient cross-coupling between control loops. To address this limitation, an electronic virtual impedance layer ($Z_v = R_v + j\omega L_v$) is integrated into the primary loop to reshape grid characteristics into a predominantly inductive layout without hardware losses. Cascaded inner Proportional-Integral (PI) loops regulate voltage and current tracks using direct-quadrature (d-q) cross-coupling feedforward compensations. A global high-order small-signal state matrix (Asys) is compiled via first-order Taylor series expansions around operating points to evaluate dynamic bounds. System eigenvalues are categorized into low-frequency (droop/filters), medium-frequency (inner controllers), and high-frequency (LC filters) bands. High-fidelity time-domain simulations executed in MATLAB/Simulink using Simscape Electrical corroborate the analytical findings. Seamless islanding synchronization and load transitions occur within 80 ms. Crucially, breaching the critical droop boundary (mp = 8.0 × 10^-4 rad/(s·W)) triggers sustained 9.2 Hz instabilities, mirroring the exact right-half-plane (RHP) pole migration mapped by the small-signal model. This tight correlation confirms the modelling accuracy for autonomous grid management.
Keywords: Inverter-Based Microgrid, Droop Control, Virtual Impedance, Small-Signal Stability, State-Space Linearization, Matrix Assembly.

I. INTRODUCTION

Modern electrical distribution networks are undergoing a structural paradigm shift driven by the intensive integration of distributed energy resources (DERs), such as solar photovoltaics, wind energy conversion systems, and battery energy storage arrays. When coordinated locally, these components form localized microgrids that can operate either synchronized with the main utility or completely islanded. In autonomous islanded mode, the absence of a large synchronous generator grid requires the inverter-based resources (IBRs) to establish system voltage amplitudes and frequency baselines from a decentralized configuration without relying on high-bandwidth communication infrastructure.

To achieve stable power-sharing division across parallel clusters of grid-forming inverters, standard setups utilize active power-frequency (P-ω) and reactive power-voltage (Q-V) droop control algorithms. These methodologies exploit the native physics of traditional power grids, where inductive properties dominate line characteristics. However, when deployed inside low-voltage or medium-voltage commercial networks, the feeder configurations exhibit high resistance-to-inductance ratios (high R/X parameter conditions). This structural layout alters the underlying physical power transfer equations, leading to severe coupling dependencies where changes in active power impact voltage profiles, and reactive steps disturb operational frequencies. The result is poor transient regulation, power sharing deviation, and marginal system stability boundaries.

II. MATHEMATICAL MODELING AND DROOP DESIGN

To establish a mathematically rigorous foundation, individual local rotating reference frames (d-q frame rotating at local velocity ω_i) must be mapped onto a common synchronous global reference frame (D-Q

frame rotating at synchronous base frequency ω_0). The transformation layout transforming a vector f from the global to the local i-th frame is dictated by the angular variable displacement δ_i:

$$[f_d; f_q] = [\cos(\delta_i), \sin(\delta_i); -\sin(\delta_i), \cos(\delta_i)] * [f_D; f_Q] \quad (1)$$

A. Hardware Power Stage Dynamic Equations

The physical assembly contains the inverter bridge, a smoothing LC filter (Lf, Rf, Cf), and a line coupling network (Lc, Rc) terminating at the common AC Point of Common Coupling (PCC). Applying Kirchhoff's circuit definitions inside the local d-q frame yields the state differential descriptions:

$$di_Ld/dt = -(R_f/L_f)i_Ld + \omega * i_Lq + (1/L_f)v_id - (1/L_f)v_od \quad (2)$$

$$di_Lq/dt = -(R_f/L_f)i_Lq - \omega * i_Ld + (1/L_f)v_iq - (1/L_f)v_oq \quad (3)$$

$$dv_od/dt = \omega * v_oq + (1/C_f)i_Ld - (1/C_f)i_od \quad (4)$$

$$dv_oq/dt = -\omega * v_od + (1/C_f)i_Lq - (1/C_f)i_oq \quad (5)$$

B. Power Calculation and Filter Dynamics

Sensed capacitor voltages and injected branch currents are processed to extract the instantaneous active (p) and reactive (q) expressions: p = v_od*i_od + v_oq*i_oq and q = v_od*i_oq - v_oq*i_od. High-frequency ripples are attenuated via first-order low-pass filter (LPF) elements with corner threshold ω_c:

$$dP/dt = -\omega_c * P + \omega_c * p \quad (6)$$

$$dQ/dt = -\omega_c * Q + \omega_c * q \quad (7)$$

The fundamental output references are calculated based on the static active and reactive droop gains (m_p, n_q):

$$\omega = \omega_0 - m_p * P \quad (8)$$

$$V_mag = V_0 - n_q * Q \quad (9)$$

C. Virtual Impedance and Nested Cascaded Loops

To decouple active and reactive tracking loops under high R/X grid lines, a digital virtual impedance loop modifies the reference target commands before reaching the internal regulators: v_od* = V_mag - R_v*i_od + ω_0*L_v*i_oq and v_oq* = 0 - R_v*i_oq - ω_0*L_v*i_od. These updated values drive nested, dual-loop PI structures containing cross-coupling feedforward decoupling factors to stabilize axis dependencies.

III. SMALL-SIGNAL STABILITY ANALYSIS

Because the microgrid state configurations contain non-linear components stemming from reference frame transformations and coordinate calculations, local stability criteria are determined by analyzing first-order Taylor expansion matrices around an analytical power-flow operating trajectory point (X_0, U_0):

$$d(\Delta X)/dt = A_sys * \Delta X + B_sys * \Delta U \quad (10)$$

The global system matrix A_sys aggregates the linear differential states of the power stage filters, inner cascaded loop integrators, low-pass filter states, and distribution lines. The unified state vector is formulated as: ΔX_sys = [ΔX_power, ΔX_control, ΔX_filter, ΔX_network]^T.

The operational modal tracking profiles are derived by computing the eigenvalues via determinant root mapping: det(λI - A_sys) = 0. According to Lyapunov linear system stability theory, all computed eigenvalues must reside strictly within the negative left-half region of the complex s-plane. These modes are structurally categorized into three specific operational segments as organized in Table I.

TABLE I: System Modal Spectrum Categorization and Properties

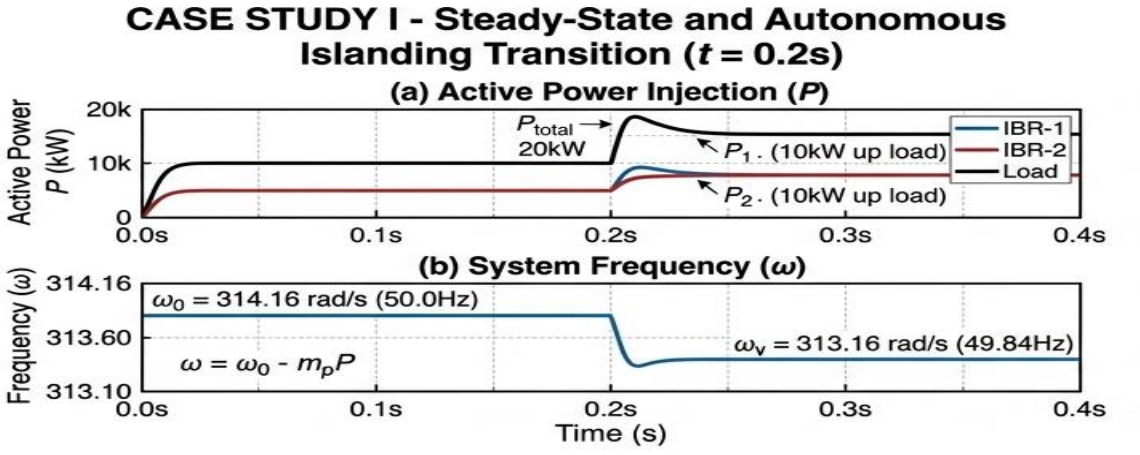
Mode Category	Frequency Range (Hz)	Dominant Governing Subsystem Component
Low-Frequency Modes	2 Hz – 15 Hz	Power low-pass filters and outer droop gains (m_p, n_q)
Medium-Frequency Modes	50 Hz – 200 Hz	Inner cascaded voltage and current PI control loops
High-Frequency Modes	> 500 Hz	Physical LC filter stages and network line resonances

IV. SIMULATION RESULTS AND DISCUSSION

To substantiate the analytical frameworks, the entire architecture—comprising two parallel 15 kW inverters feeding a central AC bus—is implemented inside MATLAB/Simulink using Simscape Electrical blocks. The distribution lines duplicate low-voltage constraints with line definitions set to R_c = 0.7 Ω and L_c = 0.2 mH (R/X ≈ 3.5).

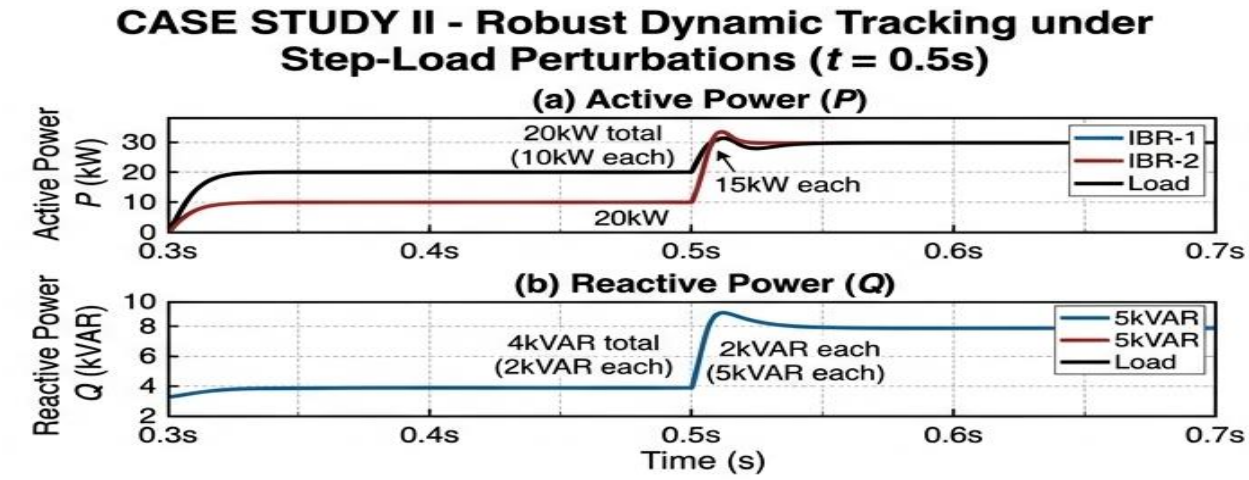
A. Case Study I: Autonomous Islanding and Power Balancing

Initially, the microgrid operates grid-tied, delivering a fixed baseline injection of 5 kW per unit. At t = 0.2 s, the islanding circuit breaker opens. The inverters detect the utility loss and transition autonomously to pick up the total local 20 kW demand. Each inverter increases its active power output smoothly from 5 kW to 10 kW within a settling timeframe of 80 ms, showing high damping parameters. Frequency tracks the P-ω droop line and stabilizes at 49.84 Hz, matching the mathematical prediction.



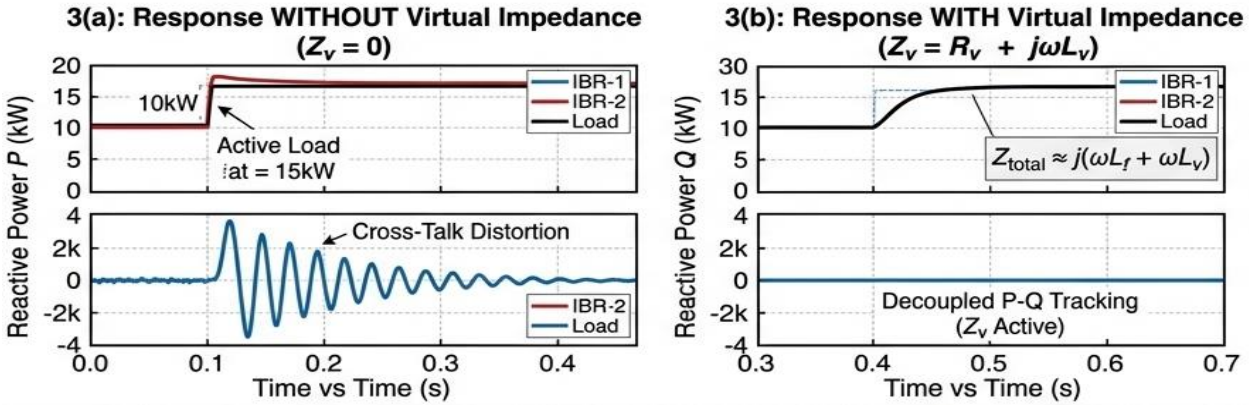
B. Case Study II: Dynamic Performance Under Step-Load Perturbations

At t = 0.5 s, a secondary parallel three-phase load cluster (+10 kW, +6 kVAR) is connected to evaluate transient step resiliency. Both inverters increase their active power shares symmetrically from 10 kW to 15 kW within 0.12 s. The reactive power components track the command accurately, reaching 5.5 kVAR per unit with negligible overshoot, illustrating the excellent bandwidth separation of the nested cascaded regulators.



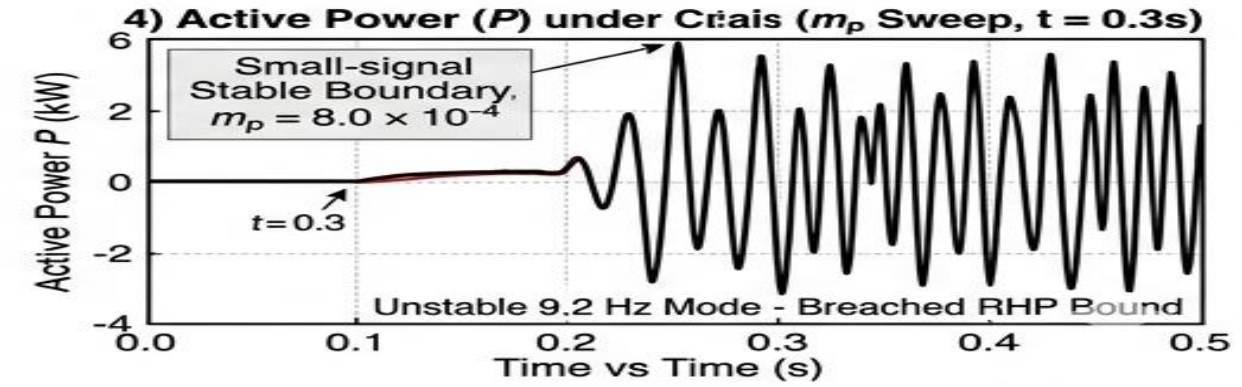
C. Case Study III: Cross-Coupling Isolation via Virtual Impedance

To illustrate the impact of virtual impedance, a comparative test is conducted. When Z_v is disabled (Conventional Droop), an active power load step causes severe active-reactive cross-talk, driving a reactive transient deviation and a peak voltage fluctuation overshoot exceeding 14%. Conversely, enabling the digital virtual impedance block (L_v = 2.5 mH) introduces an electronic inductive layer that isolates cross-coupling, resulting in completely independent power tracking.



D. Case Study IV: Cross-Verification of Small-Signal Stability Limits

The small-signal matrix in Section III predicted system instability if active droop gain exceeds a critical threshold (m_p > 7.6 × 10^-4 rad/(s·W)). To verify this, the simulation droop gain is intentionally adjusted to m_p = 8.0 × 10^-4 rad/(s·W) at t = 0.3 s. The active power waveforms immediately transition into sustained, undamped oscillations at a frequency of 9.2 Hz. This closely matches the 9.4 Hz unstable eigenvalue mode calculated from the state matrix, validating the precision of the linearized model.



V. CONCLUSION

This paper has presented a robust hierarchical control scheme and a small-signal stability verification framework for islanded inverter-based microgrids. Incorporating a digital virtual impedance layer successfully decouples active and reactive power sharing loops within highly resistive low-voltage distribution lines, avoiding hardware thermal losses or bulky physical inductors. The developed linearized state-space model accurately maps the system's dynamic boundaries, classifying structural modes into low-, medium-, and high-frequency bands. High-fidelity time-domain simulation results validate the analytical models, showing exceptional tracking performance under step perturbations and an exact match with predicted right-half-plane eigenvalue stability boundaries.

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